Towards Quantification of the need to Cooperate between Robots

K. Madhava Krishna and Henry Hexmoor
CSCE Dept., University of Arkansas
Fayetteville AR 72701

Abstract: *Collaborative technologies and reasoning strategies gain prominence with the growth in multi-agent systems, ubiquitous sensor systems and ubiquitous computing. This paper establishes the existence of a cooperative phase during real-time navigation of mobile robots where collision conflicts can be resolved only through a resort to some kind of negotiation and understanding between the robots involved. The effect of varying robot parameters on the cooperative phase is presented and the increase in requirement for cooperation with the scaling up of number of robots in a system is also illustrated.*

1. Introduction

This paper is an effort towards analyzing the need for cooperation amidst robots, which are not part of a team and hence do not share any common objectives or team goals. Specifically the requirement of cooperative strategies in the context of collision avoidance between multiple moving robots is being quantified. As a starting point for this analysis the existence of a cooperative phase during navigation in a system of two moving bodies that could resolve collision conflicts is investigated. Collision avoidance is effected purely based on velocity control alone and hence the case of head on collisions, which entails orientation control, is not considered in this endeavor.

Improved efficiency, faster responses due to spread of computational burden, augmented efficiencies and discovery of emergent behaviors that arise from interaction between individual behaviors are a few of the reasons for popularity of research in multi-robot systems. Multiple mobile robot systems find applications in many areas such as material handling operations in difficult or hazardous terrains [1], fault-tolerant systems [2], covering and guarding of unmanned terrains [3], and in cargo transportation [4]. Cooperative collision avoidance (CCA) between robots arises in all those situations where robots need to crisscross each other's path in rapid succession or come together to a common location in large numbers. Whether it is a case of cooperative navigation of robots in a rescue and relief operation after an earthquake or while searching the various parts of a building or in case of a fully automated shop floor or airports where there are only robots going about performing various chores, CCA becomes unavoidable. Possible extensions of the CCA scheme include coordination of several unmanned combat aircraft vehicles (UCAV) through a similar distributed reasoning strategy. While there has been a lot of literature on multi-robot systems we consider this to be one of the first attempts to formalize the existence of cooperation mathematically and study the requirement of cooperation in terms of parametric variations and in scaled up systems involving several robots.

Cooperation between robots becomes mandatory when solutions for individual resolution of the conflicts are exhausted in the solution space. In this particular case robots need to enter cooperation when individually they are unable to find a solution in the velocity space that could avoid the conflict (collision). A solution may not be found in the velocity space either because they do not exist or existing solutions lead to conflicts with other robots.

2. Problem Formulation:

The simple case of two robots moving in linear trajectories with constant speed is considered as a starting point of formulation. The objective of the formulation is to gather evidence for the existence of a particular phase during navigation, where robots could avoid collision by velocity control alone through a scheme of cooperation without the need for both the robots to come to a halt for averting a collision. During this particular phase called the *cooperative phase* individual resolution of conflicts (collision) would however not be possible.

Shown in figure 1, two robots R1 and R2 of radii r1 and r2 and whose states are (vc_1, vn_1, θ_1) and (vc_2, vn_2, θ_2) respectively, where vc_1, vc_2 are the current velocities while vn_1, vn_2 are the aspiring velocities for R1 and R2 respectively. Point C in the figure represents the intersection of the future paths traced by their centers. For purpose of collision detection one of the robots is shrunk to a point and the other is grown by the radius of the shrunken robot. This scenario is depicted in figure2 where R1 is depicted as a point and R2 is grown by r1 and its radius is now r1+r2.

The points of interest in figure 2 are the centers C21 and C22 of R2 where the path traced by the point robot R1 becomes tangential to R2. At all points between C21 and C22 R2 can have a potential collision with R1. C21 and C22 are at distances $(r1 + r2)\operatorname{cosec}(|\theta_1 - \theta_2|)$ on either side of C. The time taken by R2 to reach C21 and C22 given its current state (vc_2, vn_2, θ_2) is denoted by t_{21} and t_{22}. Similar computations are made for R1 with respect to R2 by making R2 a point and growing R1 by r2. Locations C11 and C12 and the time taken by R1 to reach them t_{11} and t_{12} are thus computed. A collision is said to be averted between R1 and R2 if and only if $[t_{11}, t_{12}] \cap [t_{21}, t_{22}] \in \Theta$. The locations C11, C12, C21 and C22 are marked in figure1.

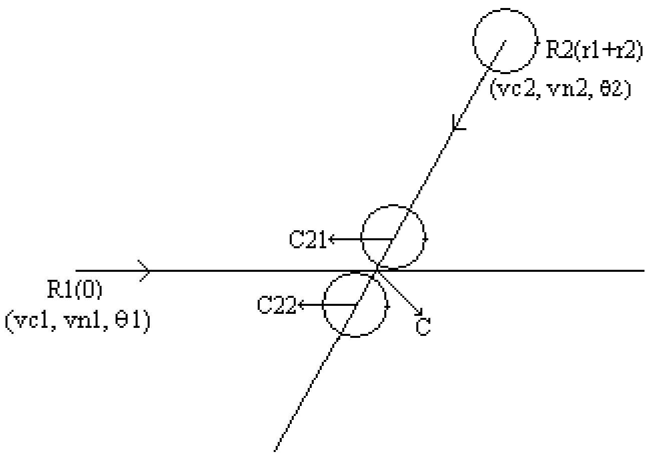

Figure 1: Two robots R1 and R2 with radii r1 and r2 along with their current states are shown

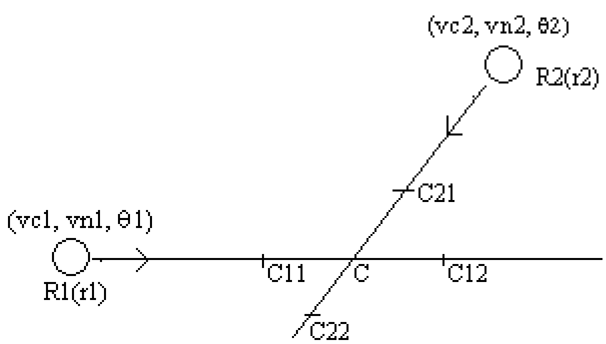

Figure 2: R1 is shrunk to a point while R2 is grown by radius of R1. C21 and C22 are centers of R2 where the path traced by R1 becomes tangential to R2.

In other words if the center of R1 occupies a space between C11 and C12 when the center of R2 lies between C21 and C22 at some time t, then collision between the two robots is deemed possible.

A collision can be averted if and only if one of the following velocity control strategies is feasible:

a. R1 does not arrive at C11 until R2 has reached C22

b. R2 does not arrive at C21 until R1 has reached C12

The velocity entailed by R1 that prevents its arrival at C11 before R2 reaches C21 under maximum deceleration, a_{-m}, is given by:

$$v_{11} \le vc_1 + a_{-m}t_{22} \pm \sqrt{(vc_1 + a_{-m}t_{22})^2 + (vc_1^2 + 2a_{-m}s)}$$

Here s denotes the distance from R1's current location to C11. In the same vein the velocity that causes R1 to be ahead of C12 when R2 reaches C21 under maximum acceleration, a_m, is given by:

$$v_{12} \ge vc_1 + a_m t_{21} \pm \sqrt{(vc_1 + a_m t_{21})^2 + (vc_1^2 + 2a_m s')},$$

where, s' the distance from R1's current location to C12 can also be written as $s' = s + (r1 + r2)\operatorname{cosec}(|\theta_1 - \theta_2|)$. In a similar fashion velocities v_{21} and v_{22} are computed.

3. Existence of a cooperative phase in robot navigation: Analysis of a two-bodied system

In the simple case of a two robot system such as above the need for cooperation arises when all the control velocities v_{11}, v_{12} for R1 and v_{21}, v_{22} for R2 do not exist in the solution space. An equivalent usage is to say that all the control velocities acquire complex values. In such a situation the robots can resort to a cooperative phase to avoid collision. In the cooperative phase one of the robots resorts to acceleration and the other resorts to deceleration. The robot that takes on an accelerative mode is the robot that reaches the center point C temporally ahead of the other. In other words if t_{1c} and t_{2c} are the time required by R1 and R2 to reach C, R1 accelerates and R2 decelerates if $t_{1c} < t_{2c}$ and vice versa.

In the multi bodied case mere existence of the control velocities does not in itself rule out the need for cooperation simply because a control velocity that enables R1 to avert collision with R2 could still result in a collision with some other robot R3. Intuitively as the number of robots increase and their navigation trajectories tend to crisscross frequently the requirement of a cooperative phase would also increase.

The description of the architecture developed for cooperative collision avoidance and the algorithms for multi robot negotiation during the cooperative phase of collision avoidance have not been dealt in this effort and are described elsewhere [5]. The focus here has been essentially to provide as close as possible a mathematical argument for the need for a cooperative phase in a multi robotic setting and also to present some empirical results that depict a relation between the requirement of cooperation vis-à-vis the number of robots in a system.

3.1 Portraying the existence through simulation:

The existence of the cooperative phase in navigation and its time span of existence vis-à-vis the angular separation between robot heading angles, $(|\theta_1 - \theta_2|)$, for the two bodied case is first presented. Robots are made to approach each other at various angular separations and the amount of solution space available for choosing control velocities that could avoid collision is computed. However the robots do not chose these velocities but continue to proceed until the solution space dries up completely indicating the onset of cooperative phase. If the robots continue to navigate without entering into a cooperative scheme for collision avoidance, a stage arises where even cooperation would not prevent collision. This final phase is termed as the destructive phase, where the robots inevitably have to collide into each other.

Figure 3 depicts a two-bodied case where the robots approach each other with an angular separation of 90 degrees. Figure 3a illustrates a graph that takes discrete values on the y-axis versus sampling instants on the x-axis. Sampling instants are those instants when the robot samples the environment through its sensor. For all the simulations portrayed in this section the time between any two successive samples is fixed

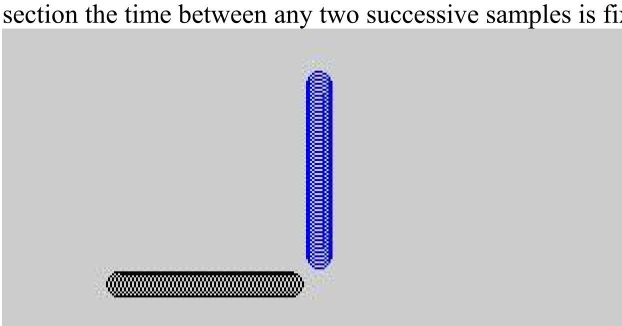

Figure 3: Two robots that approach each other with an angular separation of 90 degrees.

at 1 second, the maximum velocity of either of the robots is 5 pixels per sample and the maximum acceleration for both the robots is 2 units. The discrete values on the ordinate of figure 3 indicate the various phases of robot navigation. A ordinate value of 0 denotes what is called the *individual phase* where the robot can avoid collision individually without entering into a cooperation. Equivalently the robot is at liberty to choose control values from the solution space. A value 1 signifies the *cooperative phase* of navigation where the solution space has dried up and the robots needs to cooperate for averting collision. Finally value 2 on the ordinate implies the *destructive phase* where the robots inevitably need to collide or have already collided.

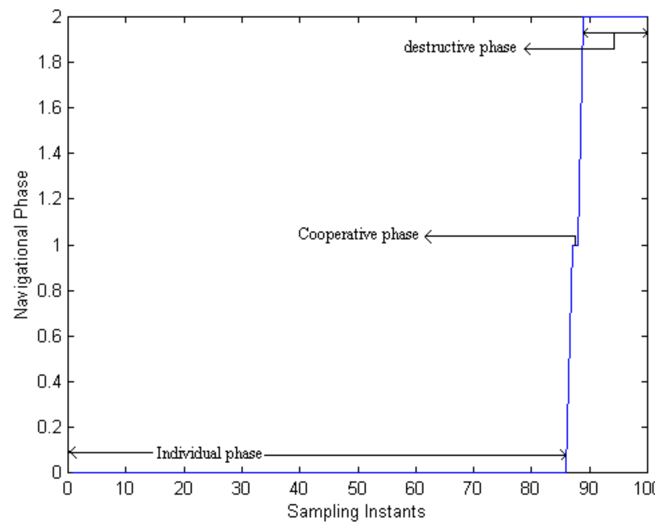

Figure 3a: The various phases of navigation versus sampling instants for an angular separation of 90 degrees between robot heading angles.

In the above figure (figure 3a) the individual phase spans for 86 sampling instants from the start of navigation while the cooperative phase extends for only two instants after which the robots enter their destructive phase.

Figure 3b depicts the percentage availability of solution space for choosing control velocities corresponding to the various navigational states of the robot in figure 3a. It is evident from figure 3b that the range of options available in the solution space decreases with time and hits zero in the 86th sample where correspondingly in figure 3a the robot enters the cooperative phase of navigation on that instant. Figures 4a and 4b depict the phases of navigation and the availability of solution space when robot pair approaches one another with an angular separation of 45 degrees, while figures 5a and 5b depict the same for a separation of 15 degrees. These figures indicate that the cooperative phase onsets earlier as the angular separation decreases and correspondingly the range of options on the solution space reduce to zero faster. The span of the cooperative phase also increases with decrease in angular separation and in figure 5a it becomes rather prominent. It is also worthwhile to note in figures 4b and 5b the percentage availability of the solution space does not overlap precisely for the robot pair over sampling instants. Hence the appearance of two distinct plots corresponding to the two robots. As a matter of fact in figure 4b the percentage availability of solution space hits zero for one of the robots ahead of the other. However the system itself enters a cooperative phase only when the solution space exhausts for both the robots. The analysis indicates that the need to resort to cooperative phase for conflict resolution would increase when robots approach one another with reduced angles of separation.

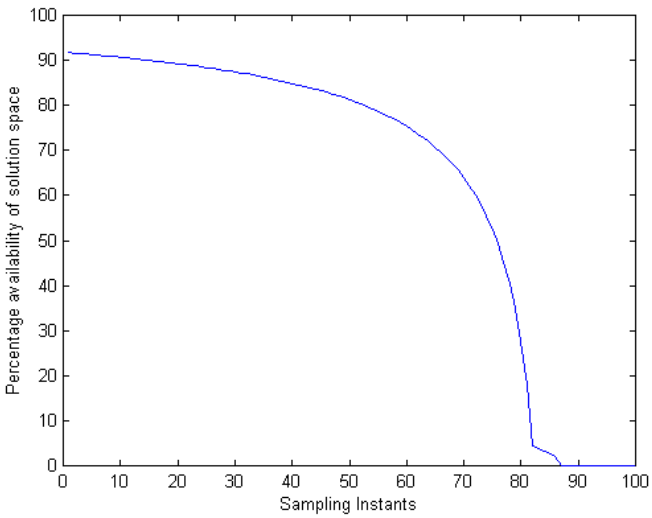

Figure 3b: Percentage availability of solution space versus sampling instants.

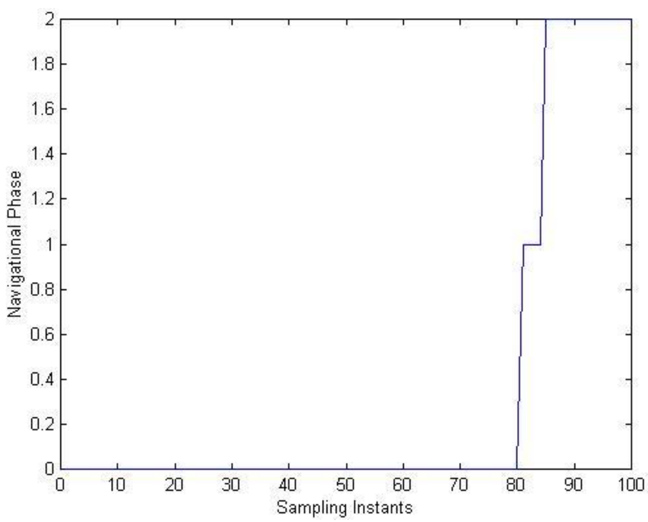

Figure 4a: Phases of navigation versus sampling instants for an angular separation of 45 degrees between robots

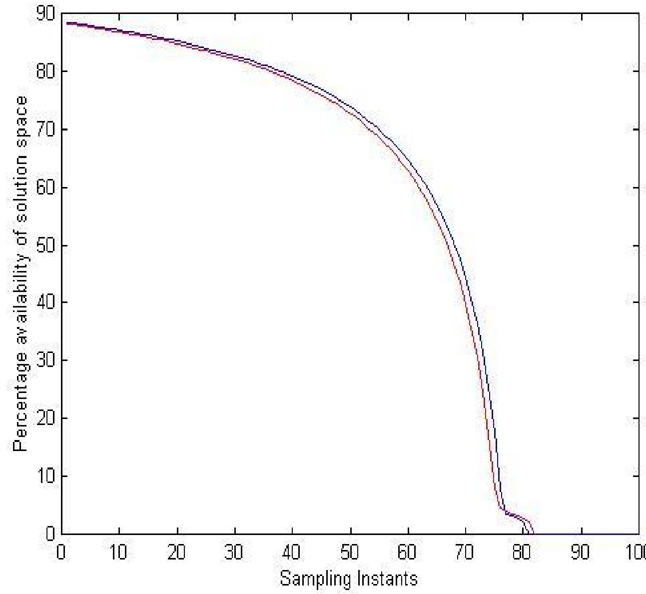

Figure 4b: Percentage availability of the solution space does not overlap precisely in this case for the two robots and hence the demarcation between the two plots.

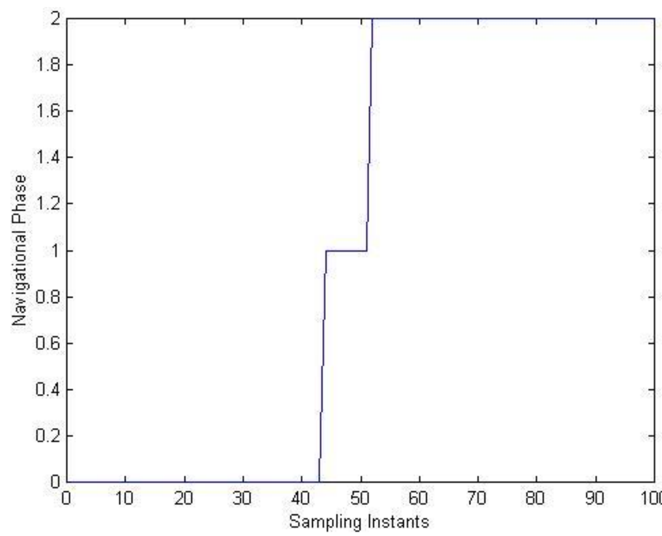

Figure 5a: The cooperative phase becomes prominent for an angular separation of 15 degrees.

4. Does existence entail requirement? When does cooperation become inevitable? :

The focus thus far has been on establishing the existence of a cooperative phase during navigation, which if resorted to could tackle the collision avoidance problem amongst moving objects. A question may be asked while the existence of a cooperative phase during navigation is not denied, how essential is the need for it.

4.1 Requirement in two-bodied system

For the two-bodied system discussed in last section cooperation could have been avoided if robots took preemptive actions before the onset of the cooperative phase. Table 1 illustrates under what set of parameters did an invocation of a cooperative scheme for collision avoidance became unavoidable. The table suggests for the case of 90 degrees separation in robot heading directions cooperation becomes inevitable only when the robot's reaction time is considerably reduced to 5seconds and when it possesses awful dynamic capabilities such as when it cannot accelerate faster or decelerate slower than $0.15 m/s^2$. However when the angular separation was 15 degrees even default parameters entailed the cooperative phase. *Hence the requirement of a cooperative scheme in real-time navigation is not artificial even for a simple two-bodied system.*

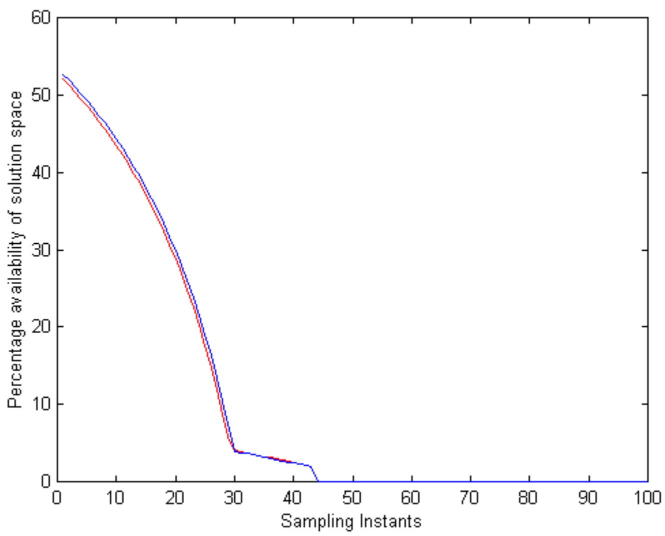

Figure 5b: Percentage availability of solution space versus sampling instants for an angular separation of 15 degrees between the robots

Angular Separation (degrees)	Reaction Time (seconds)	Maximum Acceleration, Deceleration $pixels/s^2$	Maximum velocity ($pixels/s$)
90	5	0.15,-0.15	5
45	5	0.45,-0.45	3
15	12	2,-2	1

Table 1: Robot parameters for which cooperation becomes mandatory for the two-bodied case

4.2 The multi-bodied scenario

In this subsection results from multi-bodied simulations are portrayed, where robots attempt to avoid all those collisions that are expected to occur within a given timeframe, which is called the reaction time. The reaction time was fixed at 12 seconds, the other kinematic and dynamic parameters being the same as before. In case of such large systems a technique called conflict propagation [5] is adopted to resolve conflicts when cooperation between the robots involved in the conflict alone fails to resolve it. Conflict propagation involves propagating conflicts to robots not directly involved in it but whose actions can help in resolving the conflicts between those involved. As mentioned before the details of the cooperative scheme, the architecture employed for cooperative resolution and how it coexists with the other layers in the robot's navigation architecture would not be discussed here. Figures 7 and 8 depict snapshots during navigation of a system of five and eight robots. In figure 7 cooperation was resorted once and conflict was propagated once. In figure 8 cooperation was resorted four times while conflict was propagated twice.

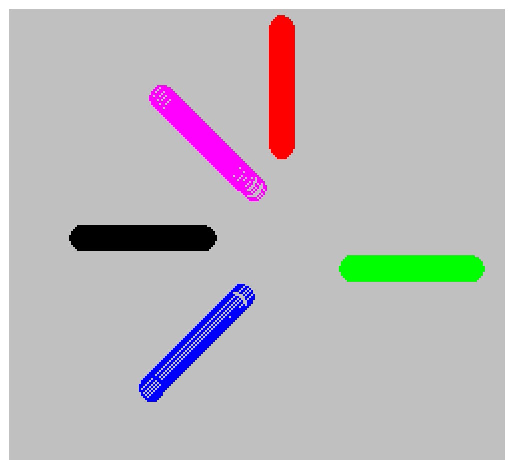

Figure 7a: Snapshot of a system of five robots

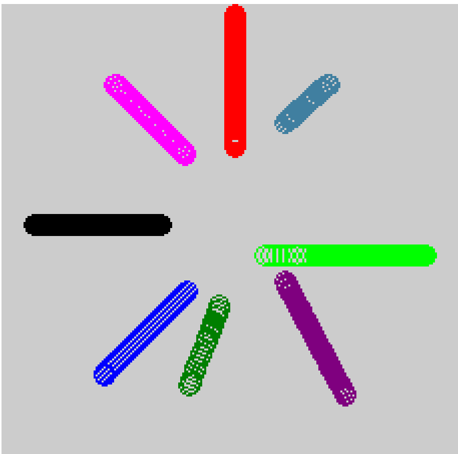

Figure 8: A snapshot of a system of 8 robots

Table 2 depicts the average number of times when cooperation and conflict propagation had to be resorted to in a system that involved large number of robots. For each system involving certain number of robots a number of runs were performed by assigning random starting and goal locations. The average number of conflicts and propagations for each such system is tabulated below. Figure 9 depicts a simulation snapshot of one such run involving 30 robots. The traces of the robots' paths are not depicted in the figure.

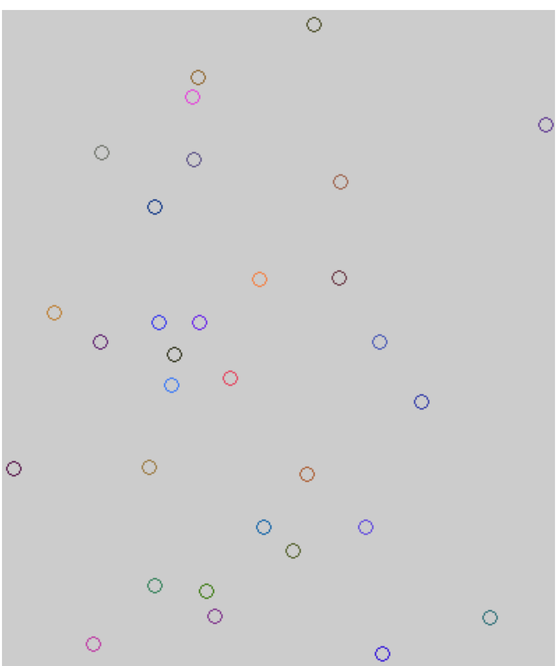

Figure 9: A system of thirty robots

The results vindicate that the need to cooperate in a multi-robotic system increase when the system scales up to a large number of robots.

Number of robots	Number of attempts at cooperation	Number of conflict propagations
10	2	2
15	4	3
20	8	4
30	12	5

Table 2: The effect of scaling up on the need to cooperate and propagate conflicts

5 Conclusions

Establishing the existence of a cooperative phase in navigation as well as ascertaining the entailment of cooperation in two robotic and multi-robotic systems involving several robots has been the contribution of this effort. Cooperative phase needs to be invoked when individual resolution of collision conflicts does not yield a control action in the individual solution spaces of the robot. Cooperation can be considered as a search for control actions (here velocities) in the joint space of the system of robots involved in conflicts. The results reported also indicate that the need to cooperate and propagate conflicts increases as the system scales up to a large number of robots.

Acknowledgements

This work is supported by AFOSR grant F49620-00-1-0302.

References:

[1] V. Genevose, R. Magni and L. Odetti, "Self-organizing Behavior and Swarm Intelligence in a Pack of Mobile Miniature Robots in Search of Pollutants", *Proc. 1992, IEEE/RSJ Int. Conf. on Intelligent Robotics and Systems,* Raleigh, NC, 1992, pp. 1575-1582

[2] L.E. Parker, ALLIANCE: An Architecture for Fault Tolerant Multi-Robot Cooperation, *IEEE Transactions on Robotics and Automation*, 14 (2), 1998.

[3] H. Choset, "Coverage for robotics - a survey of recent results". *Annals of Mathematics and Artificial Intelligence*, 31:113-126, 2001.

[4] R. Alami, S. Fleury, M. Herbb, F. Ingrand and F. Robert, "Multi Robot Cooperation in the Martha Project", *IEEE Robotics and Automation Magazine,* 5(1), 1998.

[5] K Madhava Krishna, H Hexmoor and P Subbarao, "Avoiding Collision Logjams through Cooperation and Conflict Propagation", *proceedings of KIMAS'03 (Knowledge Integrated Multi-agent Systems)*